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Rolling closed string tachyons and the big crunch

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Abstract: We study the low-energy effective field equations that couple gravity, the dilaton, and the bulk closed string tachyon of bosonic closed string theory. We establish that whenever the tachyon induces the rolling process, the string metric remains fixed while the dilaton rolls to strong coupling. For negative definite potentials we show that this results in an Einstein metric that crunches the universe in finite time. This behavior is shown to be rather generic even if the potentials are not negative definite. The solutions are reminiscent of those in the collapse stage of a cyclic universe cosmology where scalar field potentials with negative energies play a central role.

Keywords: Tachyon Condensation.

Contents

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1. Introduction

A tachyon of closed string theory is said to be a bulk tachyon if it lives throughout spacetime. In the presence of a tachyon one has an instability and two important and related questions arise:

1. Is there a ground state of the theory without the instability ?
2. What is the end-result of the physical decay process associated with the instability ?

The answers are presently known for the open string theory tachyons that live on the world-volume of unstable D-branes [1]. The ground state, or tachyon vacuum, is a state without the D-brane and without open strings — it is in fact the vacuum state of closed strings. In the associated physical decay process the D-brane dissappears but the result is not quite the closed string vacuum but rather an excited state of closed strings that carries the original energy of the D-brane. The decay process is not simply a transition from the unstable to the stable vacuum.

The purpose of the present paper is to study the physical decay induced by the bulk closed string tachyon of bosonic closed string theory — the second question above applied to bulk tachyons (for localized closed string tachyons, see [2].). Our work was prompted by new information about the first question: recently-found evidence that the closed string field theory "tachyon potential" has a critical point — a candidate for a closed string tachyon vacuum [3]. A set of considerations suggests that in such vacuum closed string states would not propagate and spacetime would cease to be dynamical. Our analysis of the physical decay aims to illuminate the nature of the tachyon vacuum. This may be possible because the physical decay turns out to be rather insensitive to the specific details of the tachyon potential, about which little is known.

We study here the low-energy field equations that couple the metric, the dilaton, and the tachyon. These equations are motivated by the conditions of conformal invariance of sigma models [4] and are expected to provide solutions that capture relevant features of exact string theory solutions. The low-energy field equations have been used in many papers to study all kinds of dynamical and cosmological issues (for a review and references see [5]). Few of these works, however, deal with key features of our present problem: a standard minimal coupling of the dilaton to other fields, an unstable closed string vacuum with zero cosmological constant, a tachyon potential that is not positive, and a rolling process induced by the tachyon. The related problem of light (bulk) tachyons that arise from circle compactification has been studied by Dine *et.al.,Dine:2003ca* and Suyama [7]. These authors compute quadratic and quartic terms in the tachyon potential and consider a cosmological evolution that involves the metric, the dilaton, the radion, and the tachyon. The authors of [6] state that numerical studies show that rather general initial conditions lead to a radius that evolves to make the tachyon more tachyonic and a dilaton that evolves to make the system strongly coupled (the simplicity of a light tachyon seems to be illusory). The author of [7] freezes the radion and discusses explicitly the simplified system, showing in a numerical solution how it appears to be driven to strong coupling.

Our analysis assumes *arbitrary* tachyonic potentials $V(T) = -\frac{1}{2}m^2T^2 + \mathcal{O}(T^3)$ and reveals a few surprises. We have found that if the rolling process is triggered by the tachyon the string metric does *not* evolve. Moreover, the dilaton expectation value Φ will always increase as time goes by. If the tachyon history $T(t)$ is such that the potential is negative, $V(T(t)) \leq 0$, the evolution reaches a singular point in finite time: both $\dot{T}$ and $\dot{\Phi}$ become infinite, and so do T and Φ. In the string frame this is a system with infinite string coupling, while in the Einstein frame the universe undergoes a big crunch. While negative potentials help accelerate its occurrance, a crunch occurs at finite time (both in the string and Einstein frames) for a wide class of potentials that grow arbitrarily large and positive for large T, $V(T) = -T^2 + T^4$, for example. The growing dilaton acts on the tachyon like anti-friction, a force proportional to the tachyon velocity in the direction of the velocity. This generally enables the tachyon to reach infinite value in finite time, even if it has to climb an infinite potential.

This paper is organized as follows. In section 2 we write the relevant coupled equations and examine them in the cosmological setting. We use the string metric and emphasize how the dilaton time derivative plays a role similar to that of *minus* the Hubble parameter $H(t)$. In section 3 we define tachyon-induced rolling and show that it results in a constant string metric. The rolling problem simplifies considerably and becomes the coupled dynamics of a dilaton and a tachyon. Analytic solutions are possible if one can solve a certain first-order nonlinear differential equation. Up to numerical constants, the dilaton-tachyon equations can be mapped to those that describe a single scalar field rolling in Einstein's theory. In section 4 we establish that the Einstein metric crunches in finite time if the tachyon potential is negative throughout the rolling solution. In section 5 we consider potentials that can be positive and develop tools to decide if there is a big crunch and if it occurs in finite time. Conclusions are offered in section 6.

2. The coupled system of rolling fields

Consider the action that describes the low-energy dynamics of the metric, the dilaton, and the tachyon:

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g}\, e^{-2\Phi} \left(R + 4(\partial_\mu \Phi)^2 - (\partial_\mu T)^2 - 2V(T) \right). \tag{2.1}$$

Here $g_{\mu\nu}$ is the string metric, Φ is the dilaton, and T is the tachyon, with potential $V(T)$. The number of spatial dimensions is d. We are following the conventions of [9], with their dilaton ϕ replaced by (-2Φ). The metric-dilaton part of the action is that in [10]. The equations of motion are:

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - (\partial_\mu T)(\partial_\nu T) &= 0\,, \\ \nabla^2 T - 2(\partial_\mu \Phi)(\partial^\mu T) - V'(T) &= 0\,, \\ \nabla^2 \Phi - 2(\partial_\mu \Phi)^2 - V(T) &= 0\,. \end{aligned} \tag{2.2}$$

To evaluate the action on-shell we multiply the first equation by $g^{\mu\nu}$, use the third equation to eliminate $\nabla^2\Phi$, and find that $R + 4\,(\partial_\mu\Phi)^2 - (\partial_\mu T)^2 = -2V(T)$. Using this,

$$S_{on-shell} = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g}\, e^{-2\Phi} \left(-4V(T)\right)\,. \tag{2.3}$$

We look for solutions of (2.2) that represent a rolling tachyon field $T(t)$ accompanied by a time dependent dilaton $\Phi(t)$ and a time dependent string metric of the form

$$ds^2 = -(dt)^2 + a^2(t) \left(dx_1^2 + dx_2^2 + \ldots dx_d^2 \right), \qquad H(t) \equiv \frac{\dot a(t)}{a(t)}\,. \tag{2.4}$$

With this metric, the gravitational equations of motion (first line in (2.2)) give two equations

$$d\frac{\ddot a}{a} + \dot T^2 - 2\ddot\Phi = 0\,, \qquad \frac{\ddot a}{a} + (d-1)\left(\frac{\dot a}{a}\right)^2 - 2\frac{\dot a}{a}\dot\Phi = 0\,. \tag{2.5}$$

The equations of motion for the dilaton and the tachyon are:

$$\ddot\Phi + \left(dH - 2\dot\Phi \right) \dot\Phi + V(T) = 0\,, \tag{2.6}$$

$$\ddot T + \left(dH - 2\dot\Phi \right) \dot T + V'(T) = 0\,. \tag{2.7}$$

We recognize the familiar Hubble "friction" term that couples H to the field velocity. Indeed, for $H > 0$ the force is opposite to the velocity and slows down the field. Similarly, the dilaton velocity $\dot\Phi$ is *anti*-friction: if $\dot\Phi > 0$ the force is in the direction of the velocity and accelerates the field. The dilaton is driven by $-V(T)$; it will tend to go to strong coupling while $V(T) < 0$.

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The gravity equations (2.5) can be rearranged into two equivalent equations:

$$\frac{1}{2}(d-1)\dot{H} = -\frac{1}{2}\dot{T}^2 + \ddot{\Phi} - H\dot{\Phi}\,, \tag{2.8}$$

$$\frac{1}{2}\,d(d-1)\,H^2 = \frac{1}{2}\dot{T}^2 - \ddot{\Phi} + dH\dot{\Phi}\,. \tag{2.9}$$

It can be shown that if equation (2.9) holds at some time, equations (2.7), (2.6), and (2.8) guarantee that it holds for all times.

It is instructive to compare of the previous equations with those that govern the dynamics of a scalar field ϕ with potential $V(\phi)$ coupled to gravity *without* a dilaton:

$$\frac{1}{2}(d-1)\dot{H} = -\frac{1}{2}\dot{\phi}^2\,, \tag{2.10}$$

$$\frac{1}{2}d(d-1)\,H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{2.11}$$

$$\ddot{\phi} + d\,H\,\dot{\phi} + V'(\phi) = 0\,. \tag{2.12}$$

Note that $\dot{H} \sim -\dot{\phi}^2 < 0$, which means decelerating expansion or accelerating contraction. On the other hand, the analogous equation in the presence of a dilaton, (2.8), allows the possibility that $\dot{H}$ vanishes. Equation (2.11) is analogous to (2.9). Comparison of (2.12) with (2.7) confirms that the rolling scalar is only affected by the addition of the dilaton-induced anti-friction ($\dot{\Phi} > 0$).

The Einstein metric $g^E_{\mu\nu}$ is determined by the string metric and the dilaton: $g^E_{\mu\nu} = \exp(-\frac{4}{d-1}\Phi)\,g_{\mu\nu}$. For a fixed string metric, the Einstein metric goes to zero if the dilaton expectation value goes to infinity. This corresponds to infinite string coupling.

3. Tachyon-driven rolling and the string metric

We now consider a general class of potentials $V(T)$ for a tachyon T that satisfy the condition $V(0) = 0$ and can be written as

$$V(T) = -\frac{1}{2}m^2T^2 + \mathcal{O}(T^3)\,. \tag{3.1}$$

We build a solution where $T \to 0$ for $t \to -\infty$, and the field rolls to positive values:

$$T(t) = e^{mt} + \sum_{n\geq 2} t_n e^{n\,mt}\,. \tag{3.2}$$

The first term in this ansatz is the solution to the linearized tachyon equation of motion. The arbitrary constant multiplying this term can be absorbed, as we did, by a redefinition of time. The exponentials in the sum are subleading to e^{mt} for large negative t. We say that the tachyon drives the rolling if the other fields, in this case $H(t)$ and $\Phi(t)$, have solutions with exponentials subleading to e^{mt}:

$$\Phi(t) = \sum_{n\geq 2}\phi_n e^{n\,mt}\,, \qquad H(t) = \sum_{n\geq 2} h_n e^{n\,mt}\,. \tag{3.3}$$

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Given (3.2), the dilaton equation (2.6) gives

$$\Phi(t) = \frac{1}{8}e^{2mt} + \mathcal{O}(e^{3mt})\,. \tag{3.4}$$

This leading behavior is valid for all potentials of the form (3.1). The dilaton begins to run towards *stronger* coupling. Evaluating the right-hand side of equation (2.8) we see that

$$-\dot{T}^2 + 2\,\ddot{\Phi} = -m^2\,e^{2mt} + 2\cdot\frac{1}{8}\cdot(4m^2)e^{2mt} + \mathcal{O}(e^{3mt}) = 0\cdot\,e^{2mt} + \mathcal{O}(e^{3mt})\,. \tag{3.5}$$

Since the other term on the right-hand side, $H\dot{\Phi} \sim e^{4mt}$, we deduce that $\dot{H} \sim e^{3mt}$ and therefore the contribution of order e^{2mt} to H vanishes: $h_2 = 0$. The string metric is not affected to this order. This is actually the beginning of a pattern: we now prove that $H(t)$ vanishes identically for tachyon-induced rolling. Adding equations (2.8) and (2.9) we find

$$\dot{H} = -(\,d\,H - 2\dot{\Phi})\,H\,. \tag{3.6}$$

Now assume that $h_2 = h_3 = \cdots = h_N = 0$ for some $N \geq 2$. Since $\dot{\Phi} \sim e^{2mt}$, the above equation gives $\dot{H} \sim e^{(N+3)\,mt}$, which implies that $h_{N+1} = 0$. By induction, $H(t)$ vanishes identically.

We now reconsider the equations of motion with $H = 0$. The gravitational equations (2.8) and (2.9) give a single equation, $\ddot{\Phi} = \frac{1}{2}\dot{T}^2$. Additionally, we have the equations of motion (2.7) and (2.6). With small rearrangements, the equations are:

$$\ddot{\Phi} = \frac{1}{2}\dot{T}^2\,, \tag{3.7}$$

$$2\,\dot{\Phi}^2 = \frac{1}{2}\dot{T}^2 + V(T)\,, \tag{3.8}$$

$$\ddot{T} - 2\,\dot{\Phi}\dot{T} + V'(T) = 0\,. \tag{3.9}$$

Since $\ddot{\Phi} \geq 0$, the dilaton velocity $\dot{\Phi}(t)$ never decreases. Given that $\dot{\Phi}(t) > 0$ for sufficiently early times (see (3.4)), the dilaton $\Phi(t)$ increases without bound. If the evolution is regular, $\Phi \to \infty$ as $t \to \infty$ (the universe takes infinite time to crunch). More generally, the evolution produces a singular point at some finite time for which, as we shall see, both $\dot{\Phi}$ and Φ become infinite. Note also the complete correspondance between the above equations and equations (2.10), (2.11), and (2.12) for an ordinary scalar coupled to gravity. The sets of equations match, up to constants, when we set $H \sim -\dot{\Phi}$. Out of the three equations above, the last two suffice. Taking the time derivative of (3.8) and using (3.9), we find that (3.7) holds as long as $\dot{\Phi} \neq 0$. The rolling of ordinary scalars with negative potentials was studied by Felder *et.al.,Felder:2002jk*, who noted that the final state is roughly independent of the shape of the potential. Given the correspondance with dilaton/tachyon rolling, this is also true in our problem.

We derived the final equations (3.9) and (3.8) using a class of initial conditions that implied $H = 0$. These equations, viewed as the original equations with the *ansatz* $H = 0$, allow more general initial conditions. For an initial time t_i we can take arbitrary $T(t_i)$ and $\dot{T}(t_i)$ as long as

$$(\dot{T}^2 + 2V(T))\big|_{t_i} \geq 0\,. \tag{3.10}$$

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The evolution is fixed by choosing a square-root branch for $\dot{\Phi}$ in (3.8). Since $\dot{\Phi}$ is positive for tachyon-driven rolling, we take

$$2\dot{\Phi} = \sqrt{\dot{T}^2 + 2V(T)}\,. \tag{3.11}$$

This enables us to rewrite (3.9) as a second-order nonlinear differential equation for the tachyon alone, an equation that is quite convenient for numerical integration:

$$\ddot{T} - \sqrt{\dot{T}^2 + 2V(T)}\,\dot{T} + V'(T) = 0\,. \tag{3.12}$$

The general rolling problem can be reduced to the problem of solving a first-order nonlinear differential equation. For this we consider the "energy" E defined as

$$E \equiv \mathcal{E}^2 \equiv \frac{1}{2}\dot{T}^2 + V(T)\,. \tag{3.13}$$

One readily checks that

$$\frac{dE}{dt} = \dot{T}\dot{T}(2\dot{\Phi})\,. \tag{3.14}$$

Since $\dot{\Phi} > 0$, E can only increase. The desired equation arises by rewriting (3.14) as

$$\frac{dE}{dt} = \pm\sqrt{E-V}\frac{dT}{dt}2\sqrt{E} \qquad \rightarrow \qquad \frac{dE}{dT} = \pm 2\sqrt{E(E-V)}\,. \tag{3.15}$$

This is an equation for $E(T)$. The sign choice arises from solving for $\dot{T}$ in terms of E and V. During evolution the sign must be changed each time $\dot{T}$ goes through zero. We will use the above mostly when $\dot{T} > 0$, so we will take the plus sign. The equation becomes a little simpler in terms of $\mathcal{E} = \sqrt{E}$:

$$\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V}\,. \tag{3.16}$$

Equipped with $\mathcal{E}(T)$, one finds $T(t)$ by solving the first-order linear equation that follows from (3.13).

A reverse engineering problem can also be solved. Suppose we are given a tachyon rolling solution specified by a function $T(t)$ that has an inverse $t(T)$. It is then possible to find the associated dilaton $\Phi(t)$ and the potential $V(T)$. We use (3.7) to find $\dot{\Phi}(t)$ by integration, and (3.8) to find $V(t)$, which gives the potential $V(t(T))$. As a simple illustration we take the leading solution in (3.2) to be exact: $T(t) = e^{mt}$. Setting integration constants to zero we find

$$T(t) = e^{mt}\,, \qquad \Phi(t) = \frac{1}{8}e^{2mt}\,, \qquad V(T) = m^2\left(-\frac{1}{2}T^2 + \frac{1}{8}T^4\right). \tag{3.17}$$

In this solution the crunch happens at infinite string time (but finite Einstein time). Related rolling solutions have been considered using two-dimensional Liouville field theory to provide conformal invariant sigma model with spacetime background fields that typically include a linear dilaton and a constant string metric [11 − 14]. In some of these solutions $T(t) = e^{mt}$ and the linear dilaton vanishes. This is unexpected given our analysis, which shows that the dilaton is sourced. It would be interesting to use this discrepancy to find constraints on the form of the effective action for the coupled system of fields.

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4. Finite-time crunch with negative scalar potentials

We now show that for non-positive potentials $V(T) \leq 0$, if $\dot{\Phi}(t_0) > 0$ for some time t_0 then $\dot{\Phi}(t_*) = \infty$ for some finite time $t_* > t_0$. To do this we combine (3.7) and (3.8) to write

$$-\frac{\ddot{\Phi}}{\dot{\Phi}^2} = -2 + \frac{V(T)}{\dot{\Phi}^2}\,. \tag{4.1}$$

Integrating both sides of the equation from an initial time t_0 up to a time t we find

$$\frac{1}{\dot{\Phi}(t)} = \frac{1}{\dot{\Phi}(t_0)} - 2(t - t_0) + \int_{t_0}^{t} \frac{V(T(t'))}{\dot{\Phi}^2(t')} dt'\,. \tag{4.2}$$

To have a divergent $\dot{\Phi}$ we need the terms on the right-hand side to add up to zero. If we ignore the integral on the right-hand side, the first two terms cancel for $t = t_1$, with $t_1 - t_0 = 1/(2\dot{\Phi}(t_0))$. Since the integral vanishes at $t = t_0$ and can only decrease afterwards, the cancellation will actually occur for a time earlier than t_1:

$$t_* \leq t_0 + \frac{1}{2\dot{\Phi}(t_0)}\,. \tag{4.3}$$

This is what we wanted to prove. It follows from (3.11) and $V \leq 0$ that as $\dot{\Phi} \to \infty$ we also have $\dot{T} \to \pm\infty$. The time evolution reaches a singular point at finite time.

To understand how the dilaton Φ itself diverges we can do an estimate that proves to be self-consistent. The integral term in (4.2) is assumed to be negligible. This is certainly the case if $V(T)$ is also bounded below since then, the integral is negligible for sufficiently large $\dot{\Phi}$. In fact, we will see that the integral is negligible under far more general circumstances. It then follows that

$$\dot{\Phi}(t) \simeq \frac{1}{2(t_* - t)} \quad \text{and} \quad \dot{T}(t) \simeq \frac{\pm 1}{(t_* - t)}\,. \tag{4.4}$$

For such solutions the tachyon and dilaton diverge logarithmically:

$$\Phi(t) = -\frac{1}{2}\ln(t_* - t) + \Phi_0 \qquad T(t) = \mp \ln(t_* - t) + T_0\,. \tag{4.5}$$

For any polynomial potential $V(T)$ the integrand in (4.2) is of the form $(t_* - t)^2 V(\ln(t_* - t))$ and goes to zero as we approach collapse, thus justifying our approximations. Note that $\dot{T}^2$ is much larger than both $V(T)$ and $V'(T)$. This fact alone implies finite time collapse: the tachyon equation (3.12) becomes $\ddot{T} \mp \dot{T}^2 \simeq 0$, whose general solution describes a $\dot{T}$ that diverges at an adjustable finite time. Note that the dilaton prefactor in the spacetime action, $e^{-2\Phi} \to e^{-2\Phi_0} \cdot (t_* - t)$, vanishes linearly with time as we approach the collapse. The string coupling becomes infinity, the Einstein metric crunches, and the value of the on-shell action (2.3) goes to zero. Since the Einstein metric becomes much smaller than the string metric as the dilaton diverges, the collapse also occurs in finite time in Einstein frame.

5. Crunching with arbitrary potentials

In this section we consider rolling solutions for rather general potentials $V(T)$, not necessarily tachyonic. As before, we take $H = 0$ and $\dot{\Phi} > 0$; these conditions ensure that we are dealing with a problem qualitatively related to tachyon-induced rolling. As a warmup we consider the case where the potential is positive and bounded and show that for a sufficiently large initial tachyon velocity the Einstein metric crunches in finite time. We then discuss a related question for more general potentials: is there an initial tachyon velocity $\dot{T} > 0$ for which $T = \infty$ (and crunching) is reached in finite time even if $V(T \to \infty) \to \infty$? We discuss a set of tools that enable one to approach this question systematically, at least in a case by case basis. We find that potentials of the form $V(T) \sim \exp(nT)$ with $n \geq 2$ are too steep, and no positive tachyon velocity allows the tachyon to reach $T = \infty$. We also analyze in detail the simple potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$.

Crunching with bounded potentials. We claim that for a bounded potential $0 \leq V(T) < \beta^2$ with bounded derivative $V'(T) < \gamma^2$, there is an initial tachyon velocity $\dot{T}(t = 0)$ for which crunching occurs in finite time. Here is a short proof. Take $\dot{T}(0) = \sqrt{\alpha^2 + 2\beta^2}$ with $\alpha^2 > \gamma^2$. Since the energy E (see (3.14)) cannot decrease in time, for $t > 0$:

$$2E(t) = \dot{T}^2(t) + 2V(T(t)) > 2E(0) = \alpha^2 + 2\beta^2 + 2V(T(0)) \geq \alpha^2 + 2\beta^2\,. \qquad (5.1)$$

Therefore, $\dot{T}^2(t) > \alpha^2 + 2\beta^2 - 2V(T(t)) > \alpha^2$ and, as a result, $\dot{T}^2(t) > \alpha^2$ for all times. The tachyon equation of motion (3.12) then gives

$$\ddot{T}(t) > \sqrt{\alpha^2 + 2V(T)}\alpha - \gamma^2 \geq \alpha^2 - \gamma^2 > 0\,. \qquad (5.2)$$

Since $\ddot{\Phi} = \frac{1}{2}\dot{T}^2$, one finds $\dddot{\Phi}(t) = \dot{T}(t)\,\ddot{T}(t) > 0$, so $\dot{\Phi}(t)$ is convex and grows without bound. On the other hand, with our bounds, equation (4.2) gives

$$\frac{1}{\dot{\Phi}(t)} < \frac{1}{\dot{\Phi}(t_0)} - \left(2 - \frac{\beta^2}{\dot{\Phi}^2(t_0)}\right)(t - t_0)\,. \qquad (5.3)$$

Since $\dot{\Phi}$ grows without bound, there is a time t_0 for which $2 - \frac{\beta^2}{\dot{\Phi}^2(t_0)} > 0$. Then at some time t_1, the right hand side of equation (5.3) vanishes. Therefore, at some finite time $t_* < t_1$, $\dot{\Phi}(t_*) = \infty$.

General techniques. We now consider a general class of potentials $V(T)$, well-defined for all T, and unbounded above as T grows positive and large. We examine an initial configuration with some fixed value $T(t_0)$ and variable initial velocity $\dot{T}(t_0) > 0$. We wish to find out if the tachyon reaches $T = \infty$ and if it does so in finite time, causing the dilaton to diverge and the Einstein metric to crunch. We find that, typically, there is a critical tachyon velocity for which it takes infinite time to reach $T = \infty$. For velocities larger than critical, $T = \infty$ is reached in finite time. For velocities smaller than critical the tachyon evolution gives a turning point.

Three curves can be defined in the $(T,\mathcal{E})$ plane and help us understand the integral curves $\mathcal{E}(T)$ that solve our first-order differentail equation $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V(T)} \equiv h(T,\mathcal{E})$:

- $h(T,\mathcal{E}) = 0$ is the *turning point* curve.
- $\frac{d}{dT}h(T,\mathcal{E}) = 0$ is the *inflection* curve. It separates a region where the integral curves are convex from a region where they are concave.
- $\mathcal{E}^2 - V(T) = f(T)$, with f specified below, is the *separating* curve. Any integral curve starting above the separating curve will remain above it.

Since $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V} = \frac{1}{\sqrt{2}}|\dot{T}(t)|$ the turning point curve is the locus of points where we get turning points for the tachyon time evolution. If an integral curve hits the turning point curve, the time evolution of the tachyon has a turning point. Moreover, since $\frac{d}{dT}\dot{T} = \frac{\ddot{T}}{\dot{T}}$, the inflection curve also controls the convexity or concavity of $T(t)$.

Consider the curve $\mathcal{E}^2 - V(T) = f(T)$. On this curve the slope of the integral curve is $\sqrt{f(T)}$. Moreover, the slope of this curve itself is $\frac{f'+V'}{2\sqrt{f+V}}$. In order to be a separating curve we require the former to be larger than the latter:

$$\sqrt{f(T)} \geq \frac{f'+V'}{2\sqrt{f+V}}\,. \tag{5.4}$$

The equality gives an integral curve — integral curves separate because they cannot cross, but are hard to find. If $V(T) \geq 0$, a suitable $f(T)$ is obtained by setting:

$$\sqrt{f(T)} = \frac{f'+V'}{2\sqrt{f}} \qquad \rightarrow \qquad 2f - f' = V'\,. \tag{5.5}$$

For a polynomial $V(T)$, a convenient choice is $f(T) = \sum_{n=1} \frac{1}{2^n}\frac{d^nV}{dT^n}$.

A worked out example. We illustrate the above discussion with the potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, which is tachyonic near $T=0$, vanishes for $T = \pm 2$, and grows arbitrarily large for large $|T|$. We consider arbitrary velocities for a tachyon for which $T=2$ for $t=0$. The potential was chosen so that it has an easily obtained solution with critical velocity: $T(t) = 2\exp(t)$. This is, in fact, the tachyon-induced rolling solution that starts at $T=0$ for $t=-\infty$. Here $T(0)=2$, $\dot{T}(0)=2$, $\mathcal{E}(T=2)=\sqrt{2}$, and the solution reaches $T=\infty$ at $t=\infty$. We have verified numerically that solutions with larger initial velocity reach $T=\infty$ in finite time, while solutions with lesser initial velocity encounter a turning point. In figure 1, we show the critical trajectory $T(t)$ and two additional solutions corresponding to initial velocities slightly higher and slightly lower than critical.

For the potential in question, the turning point curve $\mathcal{E}(T) = \sqrt{V(T)}$ lies below the inflection curve $\mathcal{E}(T) = (\frac{V+\sqrt{V^2+V'^2}}{2})^{1/2}$, which in turn lies below the separating curve ($T \geq 2$) defined with $f(T) = -\frac{1}{16}(1+2T-6T^2-4T^3)$. In figure 2 we plot these three curves along with three solutions. The solution with lowest initial energy has velocity smaller than critical: it crosses the inflection curve and hits the turning

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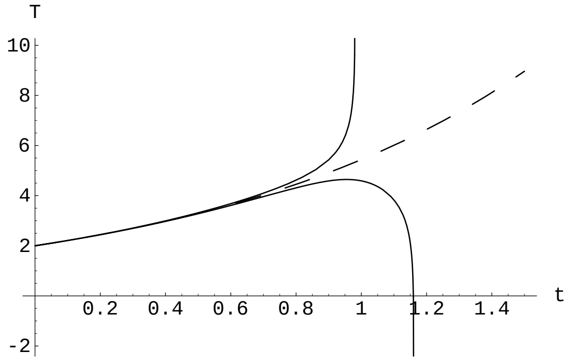

Figure 1: Tachyon histories $T(t)$ in the potential $V(T) = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, starting with $T(t=0) = 2$. The critical trajectory is dashed, begins with critical velocity $\dot{T}(0) = 2$, and crunches at infinite time. The solid line turning upwards starts with $\dot{T}(0) = 2.01$ and hits $T = \infty$ at $t \simeq 0.9808$. The solid line turning downward starts with initial velocity $\dot{T}(0) = 1.99$, encounters a turning point, and actually reaches minus infinity at $t \simeq 1.1609$.

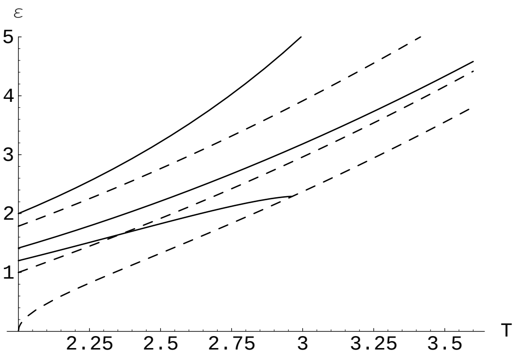

Figure 2: Curves and solutions for $\frac{d\mathcal{E}}{dT} = \sqrt{\mathcal{E}^2 - V}$ with potential $V = -\frac{1}{2}T^2 + \frac{1}{8}T^4$, plotted in the $(T, \mathcal{E})$ plane with vertical axis at $T = 2$. From bottom to top, the three dashed curves are: the turning point curve ($\mathcal{E}(T=2) = 0$), the inflection curve ($\mathcal{E}(2) = 1$), and the separating curve ($\mathcal{E}(2) = \sqrt{51}/4 \simeq 1.7854$). The three solid curves are solutions. From bottom to top they are: a solution with subcritical $\mathcal{E}(2) = 1.20$ that hits the turning point curve, the critical solution, with $\mathcal{E}(2) = \sqrt{2}$, and a solution above the separating curve, with $\mathcal{E}(2) = 2.0$.

point curve. We also show the critical solution and a solution with velocity larger than critical that lies above the separating curve. By construction any solution above the separating curve cannot have a turning point.

Potentials of the form $V(T) = \exp(nT)$**.** We now show that for $V(T) = \exp(nT)$, with $n \geq 2$, there is no initial (positive) tachyon velocity for which the tachyon can reach $T = \infty$. When $V(T) = \exp(nT)$ the differential equation (3.16) is solvable. Setting $\mathcal{E} = \sqrt{V}g(T)$, we find

$$g' + \frac{V'}{2V}g = \sqrt{g^2 - 1} \qquad \to \qquad g' = \sqrt{g^2 - 1} - \frac{n}{2}g\,. \tag{5.6}$$

We have a turning point at T_* if $g(T_*) = 1$. Dividing both sides of the equation by g and integrating,

$$\ln g(T) = \ln g(T_0) - \frac{n}{2}(T - T_0) + \int_{T_0}^{T} dT' \left(1 - \frac{1}{g^2(T')}\right)^{1/2} < \ln g(T_0) - (\frac{n}{2} - 1)(T - T_0)\,. \tag{5.7}$$

For $n > 2$, the right hand side vanishes at some $T_1 > T_0$. There is therefore some $T_* < T_1$ with $g(T_*) = 1$, and thus a turning point, as we wanted to show. The above argument in fact applies for any potential V such that $\frac{V'}{V} > 2$ for sufficiently large T. For $n = 2$, the solution of (5.6) is

$$T = T_0 + \frac{1}{2}\big(F(g) - F(g_0)\big), \qquad F(g) \equiv \ln(g + \sqrt{g^2 - 1}) - g(g + \sqrt{g^2 - 1})\,. \tag{5.8}$$

It is readily checked that $F(1) = -1$ and $F(g)$ decreases monotonically for $g > 1$. Therefore, $g(T^*) = 1$ for $T^* = T_0 - \frac{1}{2}(1 + F(g_0)) > T_0$. This proves that all solutions have a turning point.

6. Conclusions

Our analysis of tachyon-induced rolling has revealed two general facts: 1) the string metric is constant and, 2) the dilaton rolls toward stronger coupling. These facts match precisely the properties of the candidate tachyon vacuum identified in [3]. Consider fact one. In the tachyon vacuum both the tachyon and the dilaton take expectation values, and so do an infinite number of massive fields. The string metric, however, is not sourced and need not acquire an expectation value — this is guaranteed by rather general string field theory universality arguments [17, 3]. The cosmological constancy of the string metric appears to be the sigma model version of the universality result. Consider now fact two. It was shown in [3] that the dilaton expectation value in the candidate solution corresponds to stronger string coupling. The qualitative agreement makes it plausible that the rolling solutions discussed here represent rolling towards the tachyon vacuum conjectured in [3]. In the case of open strings, the end-product of the rolling solution is different but somewhat related to the tachyon vacuum. Our rolling solutions represent an Einstein metric big crunch or closed strings at infinite coupling. The tachyon vacuum may represent the dissappearance of dynamical spacetime. We feel these two states could be related. The crunch certainly lies beyond the applicability of the action (2.1), which should be supplemented by terms of higher order in α'. The generality of the evolution and the almost complete independence on the details of the tachyon potential[1] suggest to us that the cosmological solutions presented here are relevant, modulo some stringy resolution of the big crunch singularity. It would be rather interesting if the stringy resolution would push the crunch to infinite time. A big crunch, followed by a big bang, is the key element in cyclic universe models [15]. The crunch is induced by a scalar field rolling down a *negative* potential with a steep region — the rest of the potential is largely undetermined. Negative, initially steep potentials, are the hallmark of bulk closed string tachyons. We found that generally a big crunch ensues, although in our case the gravitational part of the solution is carried by the dilaton, and thus the crunch has the alternative interpretation of a closed string theory at infinite coupling. It is tempting to speculate that closed string tachyons may play a role in cyclic universe scenarios — the central difficulty remaining the mysterious transition from a big crunch to a big bang. In such studies it would be useful to focus on tachyonic heterotic models and type-0 strings.

If the vacuum of the bulk closed string tachyon truly represents the demise of fluctuating spacetime, understanding properly this state and how it fits into a consistent cosmology would give invualuable insight into the mechanisms by which a universe could come into existence. The tachyon vacuum would be roughly imagined to be the state of a universe before the Big Bang.

[1]For comments on the specific form of the tachyon potential in closed string theory see figure 1 and the conclusion section of [3].

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